



SSC-JE

STAFF SELECTION COMMISSION

CIVIL ENGINEERING

STRUCTURAL ENGINEERING

THEORY OF STRUCTURE

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CHAPTER-1:

DETERMINACY AND INDETERMINACY

Equations of Static Equilibrium

(a) Plane Structure (2D-structure): A structure is said to be 2-D structure or plane structure when all the members or forces in the structure are in one plane only.

Some examples of plane structures are:

- | | |
|-------------------|--------------------|
| (i) Beams | (ii) Plane trusses |
| (iii) Plane frame | (iv) Cables |
| (v) Arches, etc. | |

The equation of equilibrium for a planar structure are

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M = 0$$

(b) Space Structure (3D-structure): A structure is said to be 3D structures in which members and forces are in 3D.

Some examples are,

- | | |
|-----------------|-----------------------|
| (i) Space truss | (ii) Space frame etc. |
|-----------------|-----------------------|

The equations of equilibrium are:

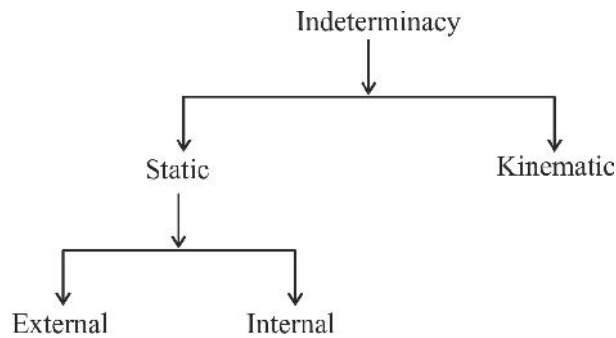
$$\begin{array}{ll} \Sigma F_x = 0 & \Sigma M_x = 0 \\ \Sigma F_y = 0 & \Sigma M_y = 0 \\ \Sigma F_z = 0 & \Sigma M_z = 0 \end{array}$$

Statically Determinate Structures:

- A structure is said to be statically determinate structure if the condition of equilibrium are sufficient to fully-analyze the structure.
- B.M. and S.F. at a section are independent of the material properties and cross-sectional dimensions of the components of the structure.
- No stresses are induced due to temperature changes and lack of fit.
e.g.-Simply supported beam, cantilever beam etc.

Statically Indeterminate Structures OR Redundant Structures:

- A structure is said to be statically indeterminate structure if the conditions of equilibrium aren't sufficient to fully analyze the structure.
- B.M. and S.F. at a section depends on the material properties and cross sectional dimensions of the components of the structure.
- Stresses are induced due to temperature changes and lack of fit.
e.g. - Fixed beam, continuous beam etc.



(A) STATIC INDETERMINACY

- If there are ‘n’ unknown like moment, shear, axial force etc. even after applying the laws of static equilibrium, the structure is said to be redundant to ‘n’ degree.
- (i) **External Redundancy:** If the external support reactions cannot be determined by using the equations of static equilibrium, the structure is termed as externally redundant.
 - It is equal to number of external reaction components in addition to number of equilibrium conditions.
- (ii) **Internal redundancy:** If the internal member forces provided to safely resist the external forces cannot be determined by using the equation of static equilibrium, the structure is termed as internally redundant.
 - It refers to geometric stability of the structure.

Total Indeterminacy: $D_s = D_{Si} + D_{Se} \rightarrow$ Degree of external static indeterminacy

Degree of internal static indeterminacy




or $D_s = \text{No. of unknown forces} - \text{Equations of static equilibrium available}$
 $D_{se} = \text{Total no. of support reactions} - \text{No. of equations of static equilibrium available}$
 $= R - 3 \rightarrow$ For 2D structure
 $= R - 6 \rightarrow$ For 3D structure

Support Reactions:

(a) Plane Structure :

	Support reactions are R_x, R_y and M_z (3 no.)
	Support reactions are R_x and R_y (2 no.)
	Support reaction is R_y (1 no.)

(b) Space Structure:

	Support reactions are $R_x, R_y, R_z, M_x, M_y, M_z$ (6 no.)
	Support reactions are R_x, R_y, R_z , (3 no.)
	Support reaction is R_y (1 no.)

Methods to determine static indeterminacy of a structure.

Case I: Pin jointed plane trusses:

Trusses are pin-jointed frames which carry only axial forces.

Static indeterminacy (SI):

$$SI = (m + r) - 2j$$

Where, m = Number of members

r = Number of reactions

j = Number of joints

➤ Total No. of unknowns = $m + r$

Number of equations available at joint = $2j$

If $SI = 0$, Structure is statically determinate

$SI > 0$, Structure is statically indeterminate

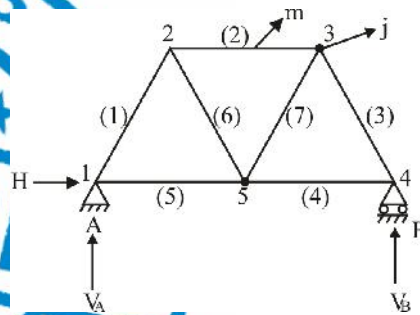
$SI < 0$, Structure is kinematically unstable

For the figure given above, the static indeterminacy (SI) is calculated as

$$S.I. = m + r_e - 2j$$

$$m = 7 \quad r_e = 3 \quad j = 5$$

$$\therefore S.I. = 7 + 3 - 2 \times 5 = 0$$



Externally redundant or indeterminate: for plane truss.

For a planar structure, there are three equations of equilibrium

$$\boxed{\sum H = 0}, \boxed{\sum V = 0} \text{ \& \ } \boxed{\sum M = 0}$$

Hence, if Number of reactions = r then,

If $r > 3$, Structure is externally indeterminate

$r = 3$, Structure is externally determinate

$r < 3$, Structure is externally Kinematically unstable

$$D_{se} \text{ (Degree of external indeterminacy)} = r - 3$$

Internally indeterminate:

If $m > (2j - 3)$, Structure is internally indeterminate

$m = (2j - 3)$, Structure is internally determinate

$m < (2j - 3)$, Structure is internally kinematically unstable.

$$D_{si} \text{ (Degree of internal indeterminacy)} = m - (2j - 3)$$

$$\text{So, } D_s \text{ (Total degree of static indeterminacy)} = D_{se} + D_{si}$$

$$= r - 3 + m - (2j - 3) = m + r - 2j$$

If a rigid frame has hybrid joints such as presence of internal hinge, link, roller etc. than some of the internal reactions will be released hence D_{si} (Degree of internal indeterminacy) will be released. If r_r is total number of released reactions then value of r_r is calculated as

$$r_r = m' - 1 \text{ \{ for 2D structure \}}$$

$$r_r = 3(m' - 1) \text{ \{ for 3D structure \}}$$

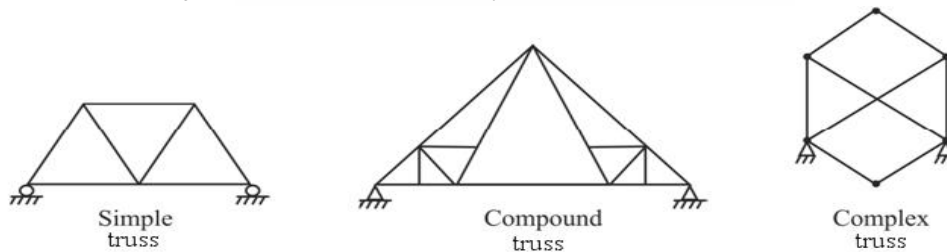
m' = number of members meeting at internal hinge.

Types of Truss

(i) **Simple truss:** When two bar and one joint are progressively added to form a truss, the truss is called simple truss.

(ii) **Compound truss:** These are the trusses formed by connecting two simple truss by a set of joints and bars.

(iii) **Complex truss:** There is no joint in the truss where only two bars meet.

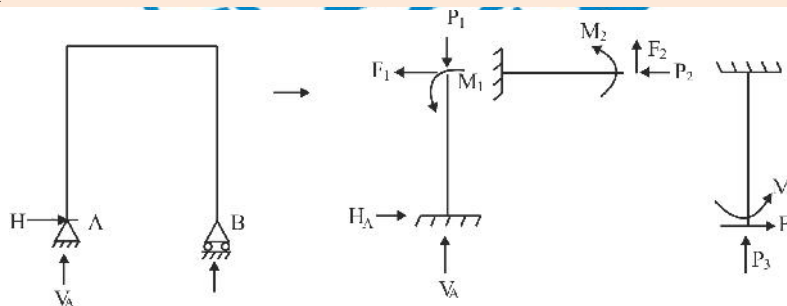


→ if hybrid joints are present then,

$$D = m + r - 2j - r'$$

r' = number of reactions released

Case II: Rigid jointed plane frame:



➤ Unlike a pin jointed frame, in rigid jointed frame, a truss member resist three stress resultant (Axial, shear force and bending moment)

Hence, [Total number of internal stresses = $3m$]

➤ Total number of unknowns = $3m + r$

➤ Also at every joint 3 equations of equilibrium are available

$$\boxed{\sum H = 0}, \boxed{\sum V = 0}, \boxed{\sum M = 0}$$

$$\boxed{\text{Total no. of equations available} = 3j}$$

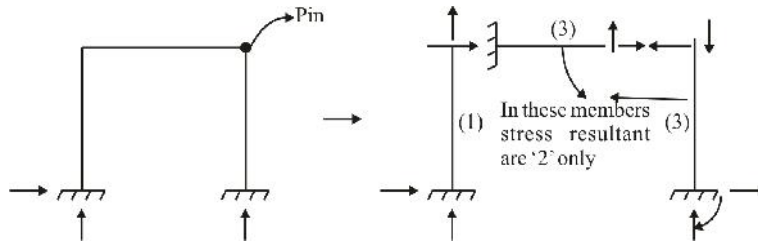
➤ Statically indeterminacy (SI)

$$\boxed{SI = (3m + r) - 3j}$$

Note: This equation cannot be used as a generalized formula for all types of frame. In such cases

$$\boxed{SI = \text{Total number of unknown} - \text{Total number of equations available}}$$

e.g.



Hence, If $SI > 0$; Structure is statically indeterminate

$SI = 0$; Structure is statically determinate

$SI < 0$; Structure is statically kinematically unstable

Externally indeterminate:

- If, $r > 3$; Structure is externally indeterminate
- $r = 0$; Structure is externally determinate
- $r < 3$; Structure is externally kinematically unstable.
- $D_{se} = r - 3$

Internally indeterminate:

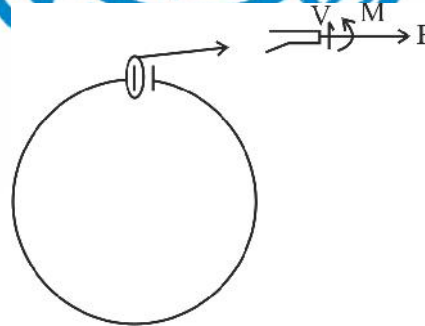
- If, $3m > (3j - 3)$; Structure is internally indeterminate
- $3m = (3j - 3)$; Structure is internally determinate
- $3m < (3j - 3)$; Structure is internally kinematically unstable
- $D_{si} = 3m - (3j - 3)$

If hybrid joints are present then, $D_s = 3m + r - 3j - r'$

Where, $r' =$ Released reactions

Ring Concept

Let us take a general plane frame member. It may be assumed as a ring subjected to loads and because of loads, it deforms. Therefore, internal forces are developed in the ring. Now, these internal forces can be found out by making a cut.



- A cut releases three internal forces, shear (V), axial force (P) and bending moment (M) at a section

Hence, total unknown member forces = 3

→Applying above concept for closed frames, static indeterminacy can be calculated as,

D_{si} (Internal Indeterminacy) = $3C - r_r$ (For 2D structure)

D_{si} (Internal Indeterminacy) = $6C - r_r$ (For 3D structure)

D_{se} (External Indeterminacy) = $R - 3$ (For 2D frames)

D_{se} (External Indeterminacy) = $R - 6$ (For 3D frames)

Where, C = Number of loops (rings)

$r_r = \sum(M'_j - 1)$ (For 2D)

$$= 3\Sigma(M'_j - 1) \quad (\text{For 3D})$$

M'_j = Number of member connecting with j' number of joints

j' = Number of hybrid joint.

Case III: Pin Jointed Space Truss :

- In a 3-dimensional pin-jointed truss, all the members carry axial force only and hence the number of total unknown internal forces = m and let total of reactions be ' r '.
Hence, total number of unknowns = $m + r$
- Number of equations available at joint
= $3j$ [$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0$, Moment equations are automatically satisfied]
- Statically indeterminacy : $SI = m + r - 3j$
- Hence, [$SI > 0, SI = 0, SI < 0$ represents statically indeterminate, determinate and Kinematically unstable structure respectively]

If hybrid joints are present then

$$D_s = m + r - 3j - r'$$

r' = number of reactions released

External Indeterminacy:

Number of equation of equilibrium is six as given by:

$$\Sigma F_x = 0$$

$$\Sigma M_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma M_y = 0$$

$$\Sigma F_z = 0$$

$$\Sigma M_z = 0$$

Hence,

- If, $r > 6$; Structure is externally indeterminate
 $r = 6$; Structure is externally determinate
 $r < 6$; Structure is externally Kinematically unstable

Internal Indeterminacy:

- If, $m > (3j - 6)$; Structure is internally indeterminate
 $m = (3j - 6)$; Structure is internally determinate
 $m < (3j - 6)$; Structure is internally kinematically unstable

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